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Value-based asset allocation: An integrated framework

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Renato Staub

is a senior risk capital analyst of William Blair's Dynamic Allocation Strategies team. Before this, he was a senior investment and risk analyst at UBS Global Asset Management in Chicago and Zurich, where he was responsible for valuation analysis, risk analysis and risk modeling, and portfolio design of liquid and alternative assets for asset allocation. He started his career in investment management in 1996 as a quantitative analyst and was involved in the development of various alternative investments. He has published articles in a variety of professional journals. He received his MS from the Swiss Federal Institute of Technology in Zurich (ETH), as well as an MA and his PhD from the University of St.Gallen – School of Management, Economics, Law, Social Sciences and International Affairs (HSG) in Switzerland.

Correspondence: Renato Staub, William Blair & Company, 222 West Adams St, Chicago, IL 60606, USA
E-mail: rstaub@williamblair.com

ABSTRACT We develop an asset allocation approach that translates valuation signals into a suggested allocation. At its core, we simulate a mean-reverting value-price evolution to infer important distribution parameters as needed in our allocation rule. The latter relies on a broad range of parameters, thereby diversifying the model risk and making the framework stable. The simulation is calibrated to meet the risk budget over time. And finally, a historical back test looks promising.

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INTRODUCTION

Pension funds face future obligations. In order to obtain commensurate returns, they invest in assets. That is, they allocate risk.

Usually, a pension fund, first, determines its long-term asset allocation. Well known as the 'policy portfolio', it comprises the long-term static mix of the fund's allocation to risky asset classes such as equity, bonds, real estate and others. The mix is supposed to meet the return target of the fund and is meant to combine risk and return properties of the various asset classes in a favorable way.

If the fund remains invested in line with its long-term strategic asset allocation, it generates the so-called passive return while being subjected to passive risk. However, the fund management may be incentivized to 'add value to the portfolio'. That is, depending on the market conditions, it moves resources from some asset classes to others.

The set of all difference positions between the effective allocation and the passive allocation is usually referenced as 'active allocation'. It is considered successful, if it results in a long-term return improvement without undue increase in portfolio risk.

In this article, we deal with active asset allocation only. That is, the policy portfolio is of no interest, although its appropriate composition is crucial as well. Furthermore, we confine ourselves to fundamental valuation. This means that our signals are based on dividend discount models only.

These help us decide whether a market is cheap or expensive. Ultimately, we are in search of a translation mechanism to convert valuation signals into an active portfolio. Although information such as momentum is certainly valuable as well, it is not considered part of fundamental investing.

To motivate this, let us consider the following example. If the US equity market is considered 10 per cent undervalued and the UK equity market 15 per cent undervalued, and if a portfolio manager actively allocates 5 per cent to US equity and 6 per cent to UK equity, how does he come up with these numbers? Of course, he has qualitative reasons embedded in a more or less appropriate gut feeling. However, again, why not 5 per cent and 7.5 per cent, or why not 5 per cent each?

In the absence of a clear answer, we feel compelled to provide the missing link. We are in search of a formal mechanism to translate the valuation signals into a suggested active portfolio. We want a clear rationale as to why a miss-valuation of X per cent results in an active allocation of Y per cent. In addition, we want the translation to be 'objective' in that it is reproducible under identical circumstances, and we want our framework to be consistent across capabilities. Furthermore, we calibrate the amount of risk to be taken in line with the given opportunity to meet the risk budget over time.

In contrast to a risk parity approach, we do not assign equal shares of portfolio risk to the various asset classes. Rather, their risk contributions are supposed to be commensurate with the embedded opportunities. These vary considerably over time. If two asset classes contribute identical amounts of risk to the portfolio, this is for reasons of coincidence.

Our game plan is the following. First, we simulate the evolutions of the individual markets' value-price (vp) discrepancies over time.[1] At each point of time, we then infer the suggested allocation corresponding to the vp dispersions in place. Finally, we investigate the properties of the implemented portfolio.

That is, given the vp signals, we are looking for the amount of active risk to be taken and the composition of the active strategy. A pivotal parameter to be identified is the scaling factor. It determines how strongly the signals must be levered into active positions such that the portfolio meets the risk budget over time.

Our examination covers various areas of expertise, such as valuation-based models, random walk modeling, implementation of mean reversion and information analysis. Ultimately, we integrate all of them into a single framework.

Our approach is rooted in 2006, and we have continuously enhanced it. Over the past four and a half years, it has been at the core of our portfolios. Two years ago, we started to implement it jointly with the New Zealand Superannuation fund for their strategic tilting program. That is, the approach is not a black box. It can rather be adjusted for individual organizations in terms of their asset universe and their own view.

The core idea of this approach and the corresponding experience in the context of the New Zealand Superannuation fund have been published most recently.[2] Although that publication portrays the approach from a high-level perspective and is centered around the 'what', the objective of this article is to focus on its technical foundation, that is, it explains the 'how'.

CENTRAL TENDENCY

A market that follows a perfect random walk does not – by definition – comprise information at all. Owing to its entirely random evolution there is no way to predict it. Although the academic point of view is that markets are efficient and hence cannot be predicted, many practitioners think that they can be predicted in the long run, at least partially.

Indeed, many years of collected data suggest that markets become disconnected at times from their fundamentals, disconnected on the upside and the downside. Markets go through boom and bust. By definition, the existence of extremes on both ends imply reversion toward the mean and beyond at times.

The contribution of this year's Nobel Prize winners in economics fits well into this discussion. Let us quote the corresponding press release from the Royal Swedish Academy of Sciences: [3]

> There is no way to predict whether the price of stocks and bonds will go up or down over the next few days or weeks. But it is quite possible to foresee the broad course of the prices of these assets over longer time periods, such as, the next three to five years.

That is, noise dominates drift in the short run, whereas drift dominates noise in the long run. This cannot apply to a pure random walk, as it has no drift. However, it applies to a mean-reverting evolution, as mean reversion is one form of drift. It is variable drift. Consistently, long-term investors tend to outperform short-term investors in the long run.

Although mean reversion cannot be proven formally, there is strong empirical evidence. To that end, let us consider the evolution of the S&P 500 price index, adjusted by the US consumer price index, over the past 20 years.

Figure 1 reveals various inflection points, the first one being the peak of the dot-com bubble in 2000, when optimism seemed unconstrained. During the subsequent sobering mode, the market plunged to half its level. Starting in 2003, it turned into a protracted uptrend, supported by cheap money. In fall 2008, the credit crisis peaked with the default of Lehman Brothers, and the market dived again to half its previous level. Ultimately, at the draught of spring 2009, the trend turned positive, interrupted repeatedly by the various stages of the European debt crisis and the US budget crisis.

On the basis of our assessment, the inflection points have been in the territory of both overvaluation and undervaluation, sometimes even considerably. However, what is ultimately responsible for such deflections? The point is that during periods of missvaluation, a market's expected and subsequently generated cash flow tend to differ, sometimes positively, sometimes negatively and sometimes markedly. If the cash flow turns out smaller (larger) than expected, this results in disappointment (goodwill), and the price that buyers are willing to pay decreases (increases). This correction mechanism forces the market price back toward intrinsic value.

A similar mechanism works in currency markets. At some point, a nation's consumption basket may become so cheap if measured in a foreign currency that it makes sense to buy goods abroad, even after

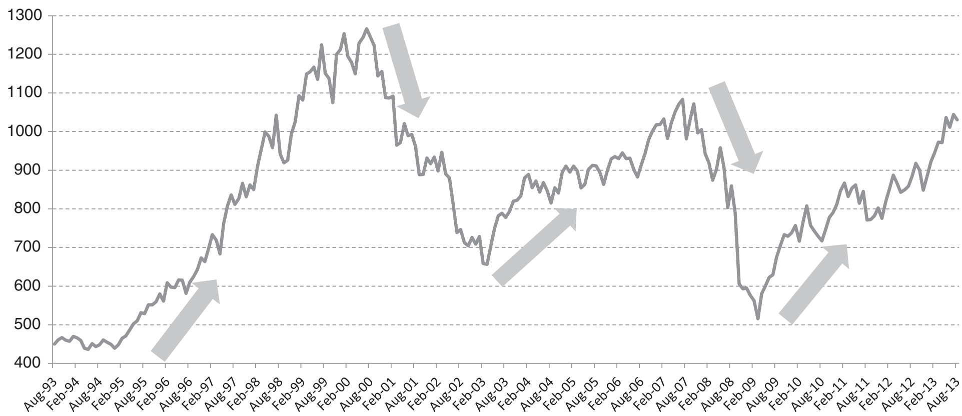

Figure 1: S&P 500 price index – CPI adjusted (08/1993 = 100).
Source: Datastream.

factoring in transaction costs. In such an environment, there is an increasing demand for the cheap currency, which makes it more expensive again. That is, the exchange rate is pulled back toward a more sustainable relationship.

SIMULATION MODEL

In the first step, we simulate the vp evolution. Note that the simulation is not about predicting the future. Rather, it is about determining calibration parameters that we need in subsequent stages.

In essence, we deal with two key inputs, a market's price, *P*, and its fundamental value, *V*. Although *P* can be observed in the market, *V* is a concept and must be estimated. Various market participants may have different perceptions of *V*.

Going forward, we simulate[4] the evolution of the logarithmic values of *V* and *P*, that is, *v* and *p*. Furthermore, we need a covariance matrix as the key source of information behind the random shocks. However, documenting the construction of a covariance matrix goes beyond the scope of this article. Even more so, as this has been documented in a publication of its own.[5]

We simulate on the basis of our long-term forward-looking covariance matrix (equilibrium matrix), which is not in line with the recent history.[6] The reason to use it nonetheless is our intention to provide a long-term examination, supposed to cover one or several entire cycles.[7]

The debate of a forward-looking versus a historical matrix in asset allocation is almost as old as asset allocation itself. Ultimately, the point is that we deal with future performance, and hence the relevant risks are future risks. Of course, risk expectations can be wrong as much as return expectations. In the end, accuracy in terms of both risk and return makes up skill.

Last but not least, a user can decide to use a short-term historical matrix in any case. May be because he faces a very short horizon or may be for another reason.

Next, to account for central tendency, we build mean reversion into the simulation. A mean-reverting market simulated in its easiest form looks as follows:

$$p_{t+1} = p_t(1 - \beta_{pp}) + \varepsilon_{t+1}$$

where β_{pp} is the mean reversion coefficient of price[8] and ε_{t+1} is a random shock. If β_{pp} equals zero, the process is a perfect random walk. If it differs from zero but its absolute value is smaller than 1, the market is mean reverting. The closer to 1, the stronger the mean reversion is.

Figure 2 shows a 1000-year vp simulation of US equity on a monthly basis. The underlying annual risk equals 15.3 per cent

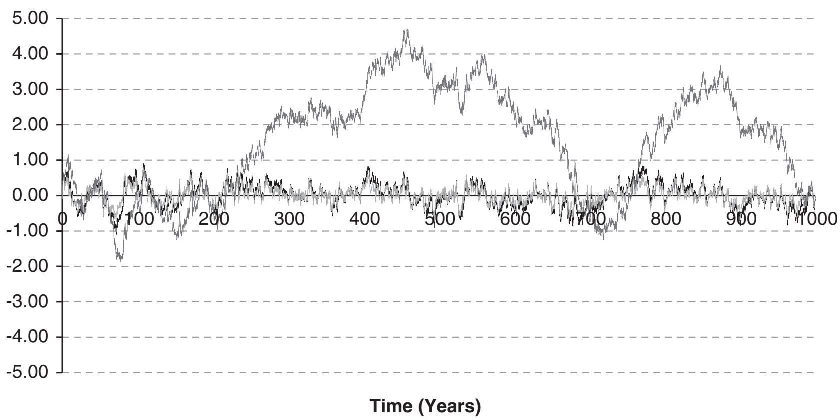

Figure 2: vp simulation of US equity on the basis of various mean reversion coefficients.
Source: proprietary.

and the mean reversion coefficients of the 3 functions are 0.00, 0.01 and 0.02.

As revealed by Figure 2, we widen the dispersion by decreasing the mean-reversion coefficient. Ultimately, in case of a zero mean reversion coefficient, the band is infinitely wide, as it represents a perfect random walk.

In case of a mean reversion coefficient of 0.01, the smallest and the largest value equal −0.75 and 0.73, corresponding to a price that equals 211 per cent and 48 per cent of the intrinsic value, which we consider realistic.[9] However, again, the decision maker of another organization may decide otherwise. He is free to do it, as much as he has to take responsibility for it.

Finally, the most deviated curve in Figure 2 shows an evolution in the absence of mean reversion. As no force pulls to the center, the vp dispersion increases with the length of the horizon. In the simulated case, we achieve the largest vp, that is, 4.71, after approximately 450 years, corresponding to a price that equals 0.90 per cent of the intrinsic value.[10] This is far from credible.

Without a doubt, as compared with the shocks, mean reversion is a minor instantaneous force. However, aggregated over time, it turns out to be a pivotal driver.[11] In the end, mean reversion is the force ensuring that vp does not defuse but stays within limits. Their breadth can be calibrated and is a key decision.

As aforementioned, it is crucial to calibrate a simulation such that the resulting distribution properties of vp are consistent with evidence. Notably, the span between the extremes on the upside and the downside is relevant. If we claimed that an equity market simulation provided extremes of 10 per cent and −10 per cent, this would be in contrast with evidence.

Although calibrating extremes is subjective and hence a challenge, we have empirical experience from the past 40 years, including bubble experience. Furthermore, given a simulation span as long as 1000 years, considerable extremes should not surprise.

Table 1 shows the simulated vp dispersions of the markets that constitute the universe of this article. Note that the ultimate simulation is somewhat more complex than explained previously. It is documented in the appendix, where the calibration is provided as well.[12]

The table has to be read as follows. In the two extreme cases, the price of the US equity

Table 1: Calibration of vp simulations

	Return(%)	*Risk(%)*	*P/V(minimum %)*	*P/V(1%)*	*P/V(50%)*	*P/V(99%)*	*P/V(maximum %)*
EQ US	7.9	15.5	33	48	101	204	248
EQ UK	7.6	15.9	37	50	100	202	332
EQ EMU	8.1	17.8	35	47	101	201	308
EQ SWI	7.4	16.6	40	53	101	182	261
EQ JAP	8.8	20.2	30	45	100	219	356
EQ AUS	8.2	16.9	44	57	99	199	288
EQ CAN	7.6	17.2	44	54	100	200	281
EQ EMA	7.9	19.2	23	38	101	257	466
BD 10Y US	5.9	7.5	72	79	100	128	150
BD 10Y UK	6.0	8.0	73	79	100	128	143
BD 10Y EMU	5.9	7.4	73	81	100	124	140
BD 10Y CH	5.7	6.4	74	83	100	122	132
BD 10Y JAP	5.8	7.1	73	79	100	121	130
BD 10Y AUS	6.0	8.7	67	78	100	132	151
BD 10Y CAN	5.9	8.3	73	80	100	130	150
BD 5Y US	5.4	4.6	81	87	100	116	127
BD 2Y US	5.0	2.4	89	93	100	108	112
HY US	5.9	7.6	64	74	100	133	158
BD EM	5.9	8.3	64	72	101	134	158
RB 10Y US	5.3	5.2	79	85	100	116	125

Source: proprietary.

market equals 33 per cent (undervalued) and 248 per cent (overvalued), respectively, of its fundamental value.

As extremes are, by nature, subject to large error margins, they are not overly stable. Hence, the 1 per cent and 99 per cent percentile are more appropriate candidates for comparison.

Although the resulting vp spans of equity markets are larger than for other asset classes because of equity markets' highest risk, they are also the largest in relative terms (that is, span versus risk). The reason is that we perceive a stronger central tendency for bonds than for equity. Consequently, we calibrate bond markets with a stronger mean reversion.

As a reference, we considered the price of the US equity market approximately 160 per cent of its intrinsic value at the peak of the internet bubble in 2000,[13] and about 50 per cent of its intrinsic value at the worst time of the financial market crisis in 2008.[14]

EMBEDDED INFORMATION

A mean-reverting time series, *s*, contains information. Mean reversion and embedded information are two sides of the same coin. As mean reversion is a balancing force, the subsequent *change* in *s* is more likely to be negative (positive) if *s* is positive (negative).[15]

In order to identify the information embedded in *s*, we calculate the

- Expected extra return[16] from reversion during the next time step;[17]
- Subsequently materialized extra return during the next time step.

The correlation between the two is a measure for the information embedded in the process. Namely, the higher the correlation, the more accurate our expectation was. The correlation is referenced as 'information coefficient' (IC). The bigger the IC, the bigger the information embedded in our vp simulation is.

We decide to model mean reversion as a linear force.[18] There is always a chance that a simulated time series moves even further out in the short run, no matter how much off-balance it already is. This is consistent with empirical evidence.

In contrast, if we decided to establish a variable mean-reversion coefficient that approaches 1 with an increasing missvaluation, this would imply that the time series could only move in one direction once having moved out far enough. In other words, it could only move back, that is, it would be deterministic.

If mean reversion is modeled as a linear force, two functions with different volatilities but identical reversion parameters have identical shapes, as one function is simply a constant multiple of the other one. Moreover, as a constant scaling factor does not add statistical information, the two evolutions have identical embedded information.

PROPORTIONAL ALLOCATION RULE

The Proportional Allocation Rule (PAR) is one of our key tools.[19] It requires to always allocate *proportionally* to our signal in order to be the most efficient. A prerequisite of PAR is that *multiple* signals should have the same volatility. Hence, we should standardize them. However, while uncorrelated signals are another prerequisite, this will, of course, not be achieved perfectly. However, the way we construct the portfolio, the correlation effect will be filtered out largely.

In the following – brief and schematic – example, let us assume that the market price moves up by Δ in a first step and back down by Δ in a second step (Table 2).

At time 0, the price equals p and the value v. We buy an amount of $(v-p)$. Next, between time 0 and 1, the price has changed by Δ. As a result, vp has changed to $(v-p-\Delta)$, that is, the discrepancy has widened. To be consistent with PAR, this requires us to adjust the quantity. In order to keep proportionality

Table 2: Schematic PAR example

Time	*Price*	*Quantity*	*Cost*
0	p	$(v-p)$	$-pv+p^2$
1	$p+\Delta$	—	—
1′	$p+\Delta$	$-\Delta$	$p\Delta+\Delta^2$
2	p	$(v-p-\Delta)$	$pv-p^2-p\Delta$
Net gain	—	—	Δ^2

Source: proprietary.

between discrepancy and quantity, we adjust the quantity by $-\Delta$; we do this instantaneously, that is, at time $1'$.[20] And finally, between time 1 and 2, the price has changed by $-\Delta$. This leaves us with the quantity ($v-p-\Delta$) at price p. The costs for buying/selling leaves us with a net gain of Δ^2over a time span of two time units.

The key to this net gain is the fact that we have a smaller exposure from price p to $p+\Delta$ (when we make a loss) than back from price $p+\Delta$ to p (when we make a gain). This example could easily be expanded to four time steps, and we would generate a gain twice as big. Furthermore, it makes no difference in which order the price movements are. That is,

- up-up-down-down
- up-down-up-down
- down-up-down-up
- and so on

lead to the same PAR performance. However, the prerequisite is a round trip, as this is what mean reversion is about. Next, we can make the time steps infinitely short and integrate the net gain over some horizon. The resulting total net gain is a linear function of the duration. That is, given a constant volatility and implementing PAR, the extra return is proportional to the time lapsed. The size of the vp extremes is not relevant.

If a market is cheap (expensive) and mean reverting, it is a 'statistical tautology' that it will outperform (underperform) in the long run. Hence, a 'proof' on the basis of a mean-reverting simulation is rather a validation of the assumptions.

ALLOCATION APPROACH

For each market,

$$xr = (v-p)/d$$

is supposed to represent the expected annual extra return because of reversion to fair value over the conversion horizon, d. Hence, the difference between two markets' extra return equals

$$\Delta xr_{ij} = xr_i - xr_j$$

This, however, is a naïve difference to be expected in case of perfect information. However, as this is not the case, the naïve expectation must be corrected by the quality of information, that is, by the embedded joint IC, and hence

$$E[\Delta xr_{ij}] = (xr_i - xr_j) \cdot IC_{ij}$$

This is the return that we can expect if we go long asset i and short asset j. The price for taking this tilt is the risk associated with this position, that is, the relative risk between asset i and asset j, σ_{ij}. That is, the units of expected return per unit of relative risk[21] equal

$$sr_{ij} = (xr_i - xr_j) \cdot IC_{ij}/\sigma_{ij}$$

We use this as our raw signal. Next, assume the following two cases. In both cases, the units of return per unit of relative risk are identical, that is, both cases are subject to the same efficiency and hence the same raw signal, which would suggest the same US$ allocation. However, assume that in the first case, i and j are bond markets, whereas in the second case, they are equity markets. That is, in the first case, this would imply a much smaller risk contribution. Hence, in order to correct this, our raw signal has to be scaled a second time by the risk distance, and we get

$$s_{ij} = (xr_i - xr_j) \cdot IC_{ij}\Big/\sigma_{ij}^2$$

This, ultimately, is our working signal. And again, according to PAR, we allocate proportionally to it, that is

$$w_{ij} = f \cdot \left(xr_i - xr_j\right) \cdot IC_{ij} \Big/ \sigma_{ij}^2$$

where f is a proportionality factor that scales to the desired level of risk. If f is doubled, the resulting portfolio risk is doubled as well.[22] The objective of the vp simulation is to identify f such that the resulting risk budget is met *ex post*.

In case of n buckets, we can make

$$\left(n^2 - n\right)/2$$

independent mutual bets. In case of 21 buckets, this would suggest

$$\left(21^2 - 21\right)/2$$

or 210 different bets. Table 3 shows all mutually relative tilts at some randomly selected point.

In this example, we go, among others, long US equity versus Canadian equity by 0.55 per cent, and in addition we go short Canadian equity versus US equity by another 0.55 per cent. In aggregate this means that we go long US equity versus Canadian equity by 1.1 per cent. Although this may sound complicated, it is easier to set up such a 'two-way street' algorithm, as we can tackle things symmetrically. Finally, the shadowed column on the right-hand end comprises the row aggregates. For instance, we would go long 4.80 per cent US equity.

With reference to its matrix structure, we call the approach 'matrix approach'. Consequently, cash is treated like all other buckets, and the resulting cash dispersion will be on the same order of magnitude as for all other buckets. This would not be the case, if all tilts were made versus cash only. In such a case, cash would literally be the ultimate 'shock absorber', and its allocation dispersion would be of a bigger magnitude than for all other buckets. In the end, tilting all markets versus cash only would result more or less in a single asset–cash bet.

Although nothing would prevent us from tilting versus cash only, the matrix approach results in a higher efficiency as it better diversifies the model risk and parameter risks throughout the entire investment universe. Moreover, as it contains all mutual tilts between equity markets, bond markets and cash, the asset–cash decision and the asset allocation decision are already embedded.

RISK BUDGET

Figure 3 shows the simulated vp of US equity. The further away the curve is from fair value, that is, from 0 per cent, the bigger the allocation to the corresponding bucket tends to be. If all markets were at fair value, there would be no reason to allocate actively.

Again, we do not run a risk parity approach, as we do not assign equal shares to risk.[23] Rather, risk goes with opportunity, and the active risk and its composition vary over time, commensurate with the opportunity.

Note that the vp evolutions have nothing to do with the scaling factor f, whose role is to determine how much the active portfolio must be levered to meet the risk budget over time.

In the given simulation, we identify the following scaling factor

$$f = 0.00222$$

It ensures that the active portfolio's risk budget of 5 per cent is met over time throughout our simulation. If we doubled the risk budget, we would have to double f as well.

Figure 4 shows the corresponding forward-looking active risk, calculated on the basis of the suggested active portfolio over time. Over a simulation span of 100 years, it varies between 2 per cent and 10 per cent, suggesting considerable opportunity swings.

How do we come up with a scaling factor of 0.00222? By trial-and-error. A bigger (smaller) scaling factor results in an overshooting (undershooting) of the risk budget. Typically, the scaling factor is identified after 3–4 iterations.

Table 3: Mutual tilts at some point during the simulation

	Equity US (%)	Equity UK (%)	Equity EMU (%)	Equity SWI (%)	Equity JAP (%)	Equity AUS (%)	Equity CAN (%)	Equity EMA (%)	Treasuries US 10Y (%)	Treasuries UK 10Y (%)	Treasuries EMU 10Y(%)
Equity US	—	0.13	−0.53	0.36	−0.07	0.29	0.55	0.30	0.21	0.45	0.27
Equity UK	−0.13	—	−0.59	0.23	−0.11	0.16	0.33	0.22	0.10	0.30	0.13
Equity EMU	0.53	0.59	—	0.64	0.27	0.67	0.86	0.53	0.46	0.60	0.51
Equity SWI	−0.36	−0.23	−0.64	—	−0.27	−0.07	0.10	0.04	−0.12	0.05	−0.09
Equity JAP	0.07	0.11	−0.27	0.27	—	0.19	0.35	0.24	0.14	0.27	0.18
Equity AUS	−0.29	−0.16	−0.67	0.07	−0.19	—	0.19	0.10	−0.04	0.13	−0.01
Equity CAN	−0.55	−0.33	−0.86	−0.10	−0.35	−0.19	—	−0.02	−0.18	−0.04	−0.18
Equity EMA	−0.30	−0.22	−0.53	−0.04	−0.24	−0.10	0.02	—	−0.11	−0.01	−0.09
Treasuries US 10Y	−0.21	−0.10	−0.46	0.12	−0.14	0.04	0.18	0.11	—	3.22	0.48
Treasuries UK 10Y	−0.45	−0.30	−0.60	−0.05	−0.27	−0.13	0.04	0.01	−3.22	—	−3.55
Treasuries EMU 10Y	−0.27	−0.13	−0.51	0.09	−0.18	0.01	0.18	0.09	−0.48	3.55	—
Treasuries CH 10Y	−0.35	−0.23	−0.57	0.00	−0.22	−0.08	0.08	0.03	−1.69	0.80	−2.77
Treasuries JAP 10Y	−0.30	−0.19	−0.51	0.03	−0.21	−0.05	0.10	0.05	−0.98	0.75	−0.65
Treasuries AUS 10Y	−0.26	−0.14	−0.53	0.09	−0.18	0.01	0.17	0.08	−0.62	1.88	−0.04
Treasuries CAN 10Y	−0.22	−0.09	−0.44	0.12	−0.15	0.05	0.22	0.11	0.19	2.54	0.52
Treasuries US 5Y	−0.30	−0.19	−0.47	0.02	−0.21	−0.05	0.10	0.04	−3.10	1.06	−1.21
Treasuries US 2Y	−0.37	−0.26	−0.51	−0.05	−0.22	−0.12	0.03	0.00	−1.66	−0.07	−1.48
High Yield Bonds US	−0.30	−0.15	−0.55	0.10	−0.18	0.01	0.18	0.10	−0.49	1.37	−0.01
EM Bonds	0.00	0.08	−0.35	0.29	−0.05	0.24	0.39	0.22	1.88	2.70	1.60
Treasuries US 10Y (Real)	−0.38	−0.26	−0.54	−0.04	−0.23	−0.12	0.04	0.01	−2.82	0.06	−1.43
Cash US	−0.38	−0.28	−0.50	−0.08	−0.26	−0.14	0.00	−0.02	−1.58	−0.26	−1.33

	Treasuries CH 10Y(%)	Treasuries JAP 10Y(%)	Treasuries AUS 10Y(%)	Treasuries CAN 10Y(%)	Treasuries US 5Y(%)	Treasuries US 2Y(%)	High Yield Bonds US(%)	EM Bonds	Treasuries US 10Y (Real)(%)	Cash US(%)	Total (%)
Equity US	0.35	0.30	0.26	0.22	0.30	0.37	0.30	0.00	0.38	0.38	**4.80**
Equity UK	0.23	0.19	0.14	0.09	0.19	0.26	0.15	–0.08	0.26	0.28	**2.35**\
Equity EMU	0.57	0.51	0.53	0.44	0.47	0.51	0.55	0.35	0.54	0.50	**10.64**
Equity SWI	0.00	–0.03	–0.09	–0.12	–0.02	0.05	–0.10	–0.29	0.04	0.08	**–2.06**
Equity JAP	0.22	0.21	0.18	0.15	0.21	0.22	0.18	0.05	0.23	0.26	**3.47**
Equity AUS	0.08	0.05	–0.01	–0.05	0.05	0.12	–0.01	–0.24	0.12	0.14	**–0.63**
Equity CAN	–0.08	–0.10	–0.17	–0.22	–0.10	–0.03	–0.18	–0.39	–0.04	0.00	**–4.12**
Equity EMA	–0.03	–0.05	–0.08	–0.11	–0.04	0.00	–0.10	–0.22	–0.01	0.02	**–2.23**
Treasuries US 10Y	1.69	0.98	0.62	–0.19	3.10	1.66	0.49	–1.88	2.82	1.58	**14.11**
Treasuries UK 10Y	–0.80	–0.75	–1.88	–2.54	–1.06	0.07	–1.37	–2.70	–0.06	0.26	**–19.37**
Treasuries EMU 10Y	2.77	0.65	0.04	–0.52	1.21	1.48	0.01	–1.60	1.43	1.33	**9.15**
Treasuries CH 10Y	—	–0.36	–0.98	–1.45	–0.39	0.79	–0.96	–2.39	0.56	0.92	**–9.25**
Treasuries JAP 10Y	0.36	—	–0.59	–0.81	0.18	0.96	–0.50	–1.86	0.79	1.05	**–2.38**
Treasuries AUS 10Y	0.98	0.59	—	–0.53	0.90	0.98	–0.01	–1.48	1.21	0.96	**4.04**
Treasuries CAN 10Y	1.45	0.81	0.53	—	1.86	1.43	0.39	–1.33	1.85	1.28	**11.13**
Treasuries US 5Y	0.39	–0.18	–0.90	–1.86	—	3.18	–1.24	–3.42	2.34	2.66	**–3.36**
Treasuries US 2Y	–0.79	–0.96	–0.98	–1.43	–3.18	—	–1.71	–3.25	–0.34	5.99	**–11.36**
High Yield Bonds US	0.96	0.50	0.01	–0.39	1.24	1.71	—	–1.66	1.55	1.59	**5.61**
EM Bonds	2.39	1.86	1.48	1.33	3.42	3.25	1.66	—	3.13	2.93	**28.45**
Treasuries US 10Y (Real)	–0.56	–0.79	–1.21	–1.85	–2.34	0.34	–1.55	–3.13	—	0.75	**–16.04**
Cash US	–0.92	–1.05	–0.96	–1.28	–2.66	–5.99	–1.59	–2.93	–0.75	—	**–22.96**

Source: proprietary.

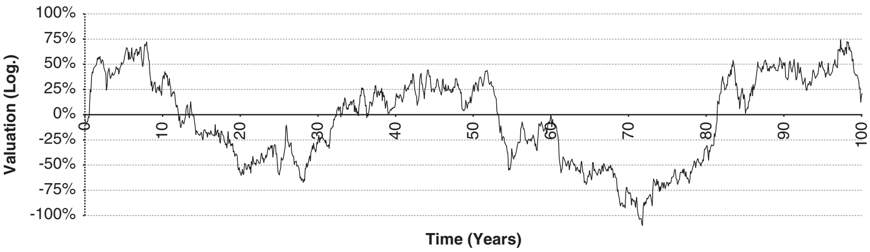

Figure 3: Simulated vp of US equity over 100 years.
Source: proprietary.

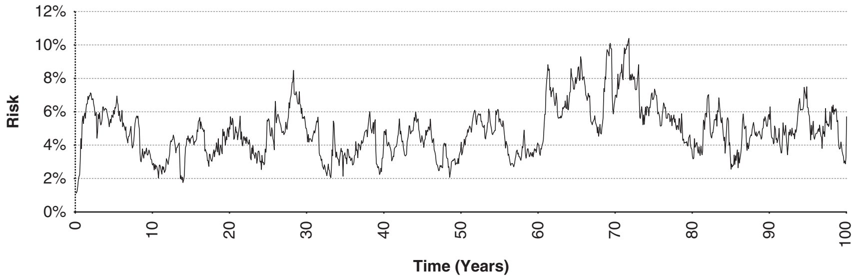

Figure 4: Forward-looking active risk based on the active portfolio over time.
Source: proprietary.

Table 4: Simulation results

Aggregate	*Return (%)*	*Risk (%)*
Cash	4.7	0.5
US equity	7.9	15.5
Base Case	6.3	5.0

Source: proprietary.

Table 4 assembles the simulation results for cash, US equity and our active portfolio, referenced as 'Base Case'.

Again, the primary objective of the simulation is a risk allocation that meets the budget over time. Once having identified *f* we can set the suggested allocation over time.

Although listed in Table 4, the resulting active return is less crucial. Of course, if negative, the simulation program would comprise an error. However, as mentioned in the previous section, simulating a portfolio on the basis of mean-reverting markets must result in outperformance. Ultimately, the relevance is the appropriate allocation of risk.

Of course, targeting the resulting active risk may be just one possible objective. For instance, we could put conditions on the total portfolio risk or on the relative amount of instances in which the total or relative risk exceed some hurdle. Furthermore, we might decide to apply allocation bounds, or we could require that the bounds became binding in no more than a certain amount of instances.

ROBUSTNESS

Our model is based on many mutual ICs. Although the *absolute* size of the ICs is irrelevant, as the portfolio is scaled by *f*, their *relative* size has an impact. In our Base Case, the largest mutual IC equals four times the size of the smallest IC.

In the following case, Case A, we make all ICs the same across the universe.[24] They equal the average size of the ICs in the Base Case, but the scaling factor is left unchanged. Figure 5 is a scatter plot of the forward-looking active risks of Case A versus the Base Case over 100 years.

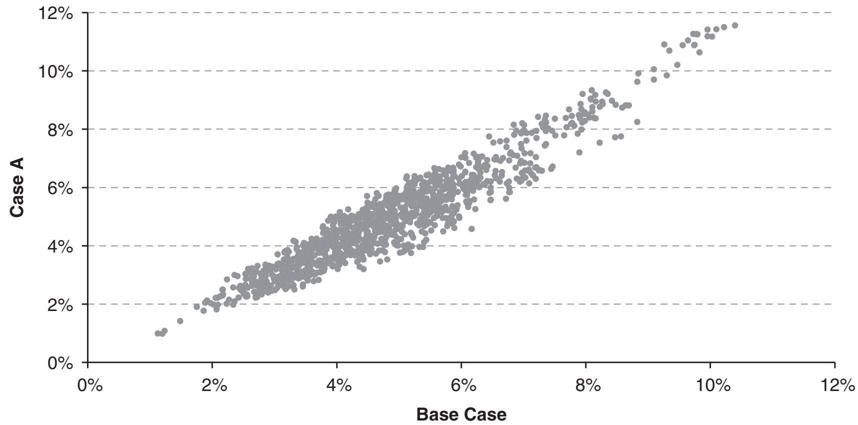

Figure 5: Scatter plot of the forward-looking active risks of Case A versus the Base Case.
Source: proprietary.

Figure 6: Scatter plot of all active risks of Case B versus the Base Case.
Source: proprietary.

The risks turn out much in line. In fact, they have a correlation of 0.95. As a result of our IC adjustment, equity–equity bets are increased, bond–bond bets are decreased and equity–bond bets are just marginally affected. However, as some individual IC scales a particular mutual tilt but never flips its sign, the tilt will never point into the opposite direction after an IC rescaling. Ultimately, the aggregate impact turns out to be very moderate.

Another important topic is the average stock–bond correlation. We think that, fundamentally, it should be positive, and this is what we have modeled into the long-term matrix. By contrast, we have observed a negative correlation over the past few years. Hence, in the next case, Case B, we simulate a universe that comprises correlations between national equity and bond markets that strictly equal −0.20.

The Figure 6 is a scatter plot of all active risks of Case B versus the Base Case over 100 years.

The scatter reveals a smaller similarity as compared with Case A, but a correlation at 0.72 is still quite high. As a result of the flipped stock–bond correlations, the risk distances between equity markets and bond markets have increased. In contrast to Case A, the equity–bond bets are more than just marginally affected by this kind of change.

Table 5: Summary of results

Case	*Return (%)*	*Risk (%)*	*Correlation with Base Case*
Base Case	6.3	5.0	1.00
Case A	5.5	5.1	0.95
Case B	6.6	6.1	0.72

Source: proprietary.

Table 5 summarizes the results of the various simulations.

In a final examination, we infer the suggested allocation at some particular point of time under the Base Case, and in a second calculation we replace the risk distances of the Base Case by the risk distances of Case B, that is, we use negative stock–bond correlations and hence bigger stock–bond risk distances. The two resulting allocations and the difference between them are provided in the Table 6.

Although the differences between individual buckets are not overwhelming, their aggregated impact is more perceptible in that there is a shift of 3.03 per cent from equity to fixed income. On the basis of the valuation signals, most mutual stock–bond tilts tend to be long equity and short bonds. However, as a result of the bigger risk distance between equity and bonds, the tilt to equity is reduced somewhat.

To conclude this section, we note that a big amount of parameters determines the entire system. It is not rooted in any single parameter driving it fairly much on its own. Rather, all parameters are involved to a similar degree. This, in turn, makes the system quite stable.

And second, we do not optimize. Optimization makes a system behave erratic at times. Moreover, correspondingly, it is a challenge at times to get the rationale behind a suggested allocation. This comes from the fact that the optimizer 'squeezes' the last basis point of return out of the system, at whatsoever cost. Hence, we often observe in an optimizing context that parameters flip all of a sudden, even after minor data changes.

Table 6: Results of final examination

Market	*Base Case (%)*	*Base Case adjusted (%)*	*Difference (%)*
Equity US	4.80	3.65	−1.16
Equity UK	2.35	1.79	−0.57
Equity EMU	10.64	8.77	−1.87
Equity SWI	−2.06	−1.79	0.26
Equity JAP	3.47	2.90	−0.56
Equity AUS	−0.63	−0.63	0.01
Equity CAN	−4.12	−3.53	0.59
Equity EMA	−2.23	−1.97	0.26
Treasuries US 10Y	14.11	14.28	0.17
Treasuries UK 10Y	−19.37	−18.74	0.63
Treasuries EMU 10Y	9.15	9.41	0.26
Treasuries CH 10Y	−9.25	−8.86	0.39
Treasuries JAP 10Y	−2.38	−2.05	0.33
Treasuries AUS 10Y	4.04	4.32	0.28
Treasuries CAN 10Y	11.13	11.27	0.14
Treasuries US 5Y	−3.36	−3.10	0.26
Treasuries US 2Y	−11.36	−11.16	0.20
High yield bonds US	5.61	5.95	0.35
EM bonds	28.45	28.14	−0.31
Treasuries US 10Y (Real)	−16.04	−15.75	0.29
Cash US	−22.96	−22.92	0.04

Source: proprietary.

We do not have to expect this kind of sign flipping in our context. Moreover, consistently, the translation of adjusted parameters into a portfolio can usually be followed in a straightforward manner. It meets common intuition.

To sum up this section, the primary objective of the simulation is a risk allocation that meets the budget over time. Once having identified *f* we can set the actual suggested allocation as based on our valuation.

EMPIRICAL VALIDATION

At the end of August 2009, we started to build suggested allocations. However, for various reasons, a historical back testing is less straight than a forward-looking simulation. First, new economic insights must be worked into the long-term forward-looking covariance matrix. Hence, we face occasional amendments of the matrix in production. Second, broader aggregates such as European Monetary Union (EMU) equities and Emerging Markets (EM) equities have been disaggregated. In addition, as the US sectors are sufficiently large to value them individually, it made sense at some point to break up US equity into its sectors. And third, every once in a while, new markets are added to the universe. The Table 7 shows the universe as grown over time.

Figure 7 presents the resulting performance index over 4 years. The annual return of 16.5 per cent goes in line with an *ex post* volatility of 22.5 per cent, resulting in an information ratio of 0.74. Again, as most markets have been mean reverting, this positive result is not a surprise by itself.

Table 7: Our universe as grown over time

Universe	
Equity Australia	Equity India
Equity Canada	Equity Indonesia
Equity France	Equity Korea
Equity Germany	Equity Malaysia
Equity Hong Kong	Equity Mexico
Equity Italy	Equity Russia
Equity Japan	Equity South Africa
Equity The Netherlands	Equity Taiwan
Equity Singapore	Treasury bonds 10Y Australia
Equity Spain	Treasury bonds 10Y Canada
Equity Sweden	Treasury Bonds 10Y Switzerland
Equity Switzerlnd	Treasury bonds 10Y Germany
Equity UK	Treasury bonds 10Y Spain
Equity US	Treasury bonds 10Y Ireland
Energy US	Treasury bonds 10Y Italy
Materials US	Treasury bonds 10Y Portugal
Industrials US	Treasury bonds 10Y Greece
Consumer discretionary US	Treasury bonds 10Y Japan
Consumer staples US	Treasury bonds 10Y US
Financials US	Treasury bonds 10Y UK
Health care US	Treasury bonds 10Y US (ILG)
Information Technology US	High yield bonds US
Telecom services US	Treasury bonds EM (USD)
Utilities US	Corporate bonds US
Equity Brazil	Mortgage backed securities US
Equity China	Treasury bills 6M US

Source: proprietary.

However, this is not the end of the discussion yet. First, it is important to understand that strictly implementing a fundamental approach does not imply – by definition – that performance will be smooth. If most equity markets are undervalued, this will result mainly in equity long positions. However, it is possible that equity prices fall even further. Market participants may become temporarily risk averse for a multitude of possible reasons. Consequently, they discount markets by bigger risk premia.

This means that the opportunity becomes even bigger. Hence, in this environment, it is paramount to stick with PAR. The third year of our back test, which looks dissatisfactory, falls into this category. The European debt crisis and the US budget crisis were mainly responsible for this drop, but at the same time they created further opportunity. From our point of view, the Spanish and Italian equity markets, for instance, are still massively undervalued.

Finally, the resulting historical portfolio volatility turns out about twice as high as targeted in the long run. On the other hand, it is important to acknowledge that the past few years were a time of big opportunity, and our approach takes risk commensurate with opportunity. On the other hand, we will not waste the risk budget, if we spot no opportunity. Hence, inferring from probably less than half a cycle to the entire cycle is

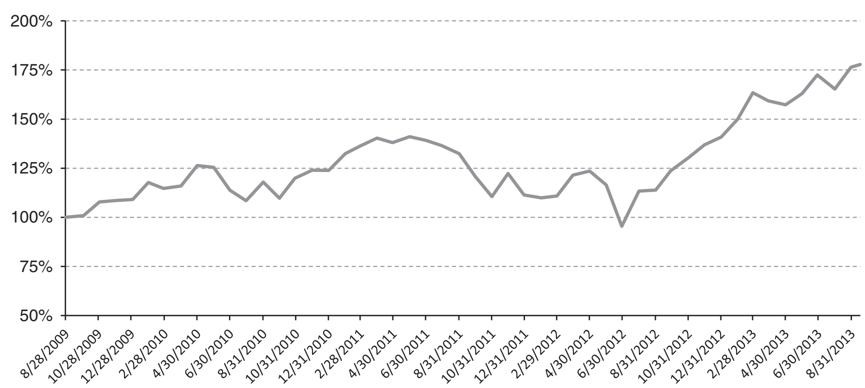

Figure 7: Performance index of the model back testing.
Source: proprietary.

inappropriate. The full cycle will entail protracted periods of sub-par opportunity and hence we will take much less risk.

ACTUAL ALLOCATION

Table 8 shows the suggested allocation, given the calibration as inferred throughout this article and our valuation as of late. Although fixed income is considered overvalued almost across the board, there are both undervalued and overvalued equity markets.

This results in a relative risk of 6.9 per cent, which is larger than its long-term target of 5 per cent. Given the massive overvaluation of most fixed income markets, combined with a very strong undervaluation of UK equity and EMU equity, this outcome makes sense.

The given allocation reveals a characteristic feature of the matrix approach. Namely, an undervalued (overvalued) market may result – nonetheless – in a negative (positive) allocation. The reason is that a market may be considered undervalued (overvalued) but to a lesser degree than many other markets.

Table 8: Suggested allocation, given some recent valuation

Market	*P/V (%)*	*Suggested allocation (%)*
Equity US	103	1.13
Equity UK	70	11.57
Equity EMU	66	10.17
Equity SWI	105	0.66
Equity JAP	101	0.87
Equity AUS	89	4.54
Equity CAN	98	1.94
Equity EMA	84	3.40
Treasuries US 10Y	119	−20.12
Treasuries UK 10Y	120	−18.18
Treasuries EMU 10Y	120	−20.10
Treasuries CH 10Y	119	−17.71
Treasuries JAP 10Y	125	−28.15
Treasuries AUS 10Y	112	3.45
Treasuries CAN 10Y	123	−26.87
Treasuries US 5Y	110	9.46
Treasuries US 2Y	104	12.84
High yield bonds US	100	32.32
EM bonds	105	13.49
Treasuries US 10Y (Real)	114	−7.97
Cash US	101	33.27

Source: proprietary.

The table contains various examples for this. Although Australian 10Y bonds are considered overvalued, our suggested allocation to them is, nonetheless, positive. The point is that there are several other fixed income markets that are considered substantially more overvalued. This may lead to multiple mutual tilts in which Australian bonds take a long position, which may result in a positive aggregate Australian bond position, no matter whether Australian bonds are expensive.

SUMMARY AND CONCLUSIONS

Our primary objective is a risk allocation that meets the budget over time. To that end, we develop an asset allocation approach that translates valuation signals into a suggested allocation. The approach is supposed to be transparent and consistent across markets.

At its core, we simulate a mean-reverting vp evolution. A simulation is a straight way to infer necessary distribution parameters, which will be used when it comes to setting the allocation as based on the present valuation.

In a subsequent step, we extract the signals from the simulated vp evolution and infer all mutual tilts possible between any two markets. We set a mutual tilt on the basis of the difference between the two involved extra returns. An extra return is a market's expected compensation above or below its fair compensation.

As a result of the matrix structure of these tilts, we call the approach 'matrix approach'. Not only does it perform better than an approach that makes all tilts versus cash only, but the resulting cash allocation is more stable as well. Ultimately, cash is treated like any other bucket.

The matrix approach performs well, as it better diversifies the model risk and parameter risks throughout the entire system. In the end, many parameters determine the entire system. The approach is not rooted in any single parameter, driving it fairly much on its own. This makes our framework quite stable.

In addition, because it establishes all mutual bets between equity markets, bond markets and cash, the asset–cash decision and the asset allocation decision are embedded already.

Last but not least, a historical back test of the approach looks promising, although it covers less than probably half a cycle at this time.

NOTES

1. V and P are measured as regular numeraires. That is, their minimum value equals zero, and their maximum is unlimited. The 'true' intrinsic value is defined as 1. Furthermore, the lower case labels, v and p, define the logarithmic values of V and P.
2. See Iverson and Staub (2013).
3. See Jarnestad (2013); Fama (1970 and 1991); Campbell and Shiller (2007) and Shiller (2000).
4. This process is documented more formally in Appendix A.
5. See Staub (2006).
6. The matrix is provided in Appendix C.
7. For our own production, we have developed a framework that is based on volatile risk parameters. That is, risks and correlations vary. We calibrate them such that their average size over a full cycle is in line with our long-term estimates. Again, this approach goes way beyond the scope of an introduction.
8. As we will show in the appendix, we may think as well of a mean reversion versus another series.
9. $\mathrm{vp} \equiv v{-}p = \log(V/P)$; $v{-}p = V/P = \exp(-0.75)$ and $V/P = 0.4724$ or $P = V/0.4724 = V{-}2.1170$.
10. $\mathrm{vp} \equiv v{-}p = \log(V/P) = 4.71$; $V/P = \exp(4.71)$; $P = V/111.05$.
11. As the aggregation of mean reversion works, unlike random shocks, repeatedly in the same direction.
12. See Appendix A.
13. Note, while the technology sector was overvalued massively, the overvaluation of the US equity market without technology was less extreme.
14. This was our assessment when we were at UBS Global Asset Management.
15. s is considered in log space.
16. The extra return is the difference between the expected compensation and the fair compensation.
17. In order to calculate the expected extra return, we must assume a reversion time. However, the reversion time does not impact the subsequent correlation calculation, as long as it is *constant*.
18. More clearly, a linear force in log space, as we model everything in log space.
19. See Staub (2007, p. 369f).
20. Time 1 and 1′ are meant to be two subsequent points of time that are infinitely close.
21. It is the same concept as the Sharpe Ratio.
22. This, however, only applies to an unconstrained portfolio. If a portfolio is constrained and already has a high risk, it is increasingly impossible to double its risk.
23. See Callan Associates (2010, p. 11).
24. This is a typical suggestion from practitioners, as they want nothing 'magic' behind the IC dispersion.
25. That is, v is not an 'objective value'; rather, it is our perception.

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APPENDIX A:

Appendix A: vp Modeling

In the first step assume that the price follows a random walk. That is

$$p_{t+1} = p_t + \varepsilon_{t+1}$$

where ε_{t+1} is a random shock. Typically, p_{t+1} contains a series of markets. Hence, ε_{t+1} is not a scalar but a vector of random shocks. These are specified in the covariance matrix. Furthermore, as a perfect random walk defuses, we must ensure *mean reversion*. Hence, we adjust the above equation as follows

$$p_{t+1} = p_t\left(1-\beta_{pp}\right)+\varepsilon_{t+1}$$

where β_{pp} is the of mean reversion of the price. From time series analysis, we know that the transition from

$$\beta_{pp} <= 0$$

to

$$\beta_{pp} > 0$$

marks the transition from a non-stationary to a stationary process.With regard to v, the point is that nobody knows it, and this is why we must estimate it. In our model, we assume that our estimate of the intrinsic value fluctuates around the 'true' intrinsic value that nobody knows. Hence,

$$v_{t+1} = v_t(1-\beta_{vv})+\eta_{t+1}$$

where β_{vv} is the mean reversion of the value.[25] Furthermore, there is evidence that models are reviewed more often in case of large vp discrepancies. That is, we tend to question the model rather than staying course. Such behavior applies in particular to those markets in which we have low model confidence, and the corresponding reviews typically narrow the discrepancy.

Technically, this means that there is a gap sensitivity of the assumed v versus p, that is, the perceived value mean reverts around price as well. Such behavior is not necessarily surprising, as it happens in case of strong momentum, that is, most participants think that the price is justified. We refer to this phenomenon as 'chasing'; the value chases the price. Algebraically, this means

$$v_{t+1} = v_t(1-\beta_{vv})+(v_t-p_t)\left(1-\beta_{vp}\right)+\eta_{t+1}$$

or

$$v_{t+1} = v_t\left(2-\beta_{vv}-\beta_{vp}\right)-p_t\left(1-\beta_{vp}\right)+\eta_{t+1}$$

where β_{vp} is the gap sensitivity of mean reversion of value versus price. Owing to chasing, the average opportunity becomes

- More of a perception than reality
- Smaller

Table A1: Calibration parameters of the vp simulation

Bucket	*Market Risk %*	*Valuation Risk %*	*mv*	*ch*
EQ US	15.3	3.8	0.005	0.005
EQ UK	15.9	4.0	0.005	0.005
EQ EMU	17.6	4.4	0.005	0.010
EQ SWI	16.7	4.2	0.005	0.010
EQ JAP	20.0	5.0	0.005	0.010
EQ AUS	16.7	4.2	0.005	0.010
EQ CAN	17.0	4.3	0.005	0.010
EQ EMA	19.1	9.5	0.003	0.010
BD 10Y US	7.4	1.8	0.020	0.005
BD 10Y UK	8.0	2.0	0.020	0.005
BD 10Y EMU	7.4	1.8	0.020	0.005
BD 10Y CH	6.4	1.6	0.020	0.005
BD 10Y JAP	7.1	1.8	0.020	0.005
BD 10Y AUS	8.6	2.2	0.020	0.005
BD 10Y CAN	8.2	2.1	0.020	0.005
BD 5Y US	4.6	1.1	0.020	0.005
BD 2Y US	2.3	0.6	0.020	0.005
HY US	7.5	3.7	0.010	0.010
BD EM	8.1	4.0	0.010	0.010
RB 10Y US	5.1	1.3	0.020	0.005
SB US	0.4	0.1	0.080	0.010

Source: proprietary.

Ultimately, after subtracting price from value, we get

$$vp_{t+1} = v_t\left(2-\beta_{vv}-\beta_{vp}\right)-p_t\left(2-\beta_{pp}-\beta_{vp}\right) + \eta_{t+1}-\varepsilon_{t+1}$$

Setting the final calibration, we generally assume less uncertainty in valuations of more developed markets. Without a doubt, the S&P 500 is more examined than the Indonesian equity market, which is part of EM equity in the Table A1. Consequently, we assume less uncertainty in the valuation of the S&P 500.

Furthermore, we assume stronger mean reversion for developed markets and more mean reversion for fixed income than for equity. And finally, chasing tends to be stronger for less investigated markets. In such cases, there is a bigger tendency to confirm the market price by the valuation model. As a result, the opportunity tends to be perceived smaller than the real opportunity.

Although Table A1 shows the calibration underlying our vp simulation, we need to acknowledge that calibration elements cannot be 'proven' on the basis of theoretical considerations. Setting them is largely a question of common sense, and this involves much backward calibration.

APPENDIX B:

Appendix B: PAR – Numerical example

As PAR is a central element of our approach, it is worthwhile to present the way it functions more visually. As its name expresses, it is the idea of PAR to persistently allocate proportionally to the signal.

Figure B1 shows the easiest representation of PAR. Assume a fairly priced market. Consequently, we have no active position. The market moves one unit away from fair value; in our case it becomes cheaper, that is, it is now underpriced by one unit. Accordingly, we buy one unit. In a second step, the market gets more expensive again by one unit. That is, it moves back to fair value. Now, we could sell our unit at a higher price than we bought it. Overall, we are better off by one unit than before.

Again, the trick of PAR is that the market moves first and we adjust thereafter. This example can easily be expanded to four time steps. Then we would generate a gain twice as big. The only prerequisite is a round trip, which is another characterization of mean reversion.

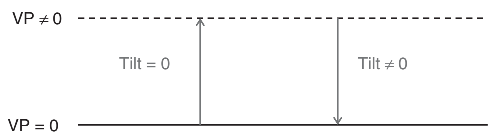

Figure B1: Mechanism of PAR.
Source: proprietary.

Figure B2 introduces a 'triangle sine' function. Initially, there is a growing vp discrepancy that peaks at the first quarter of the cycle, then it decreases, and after the draught at three quarters of the cycle, it moves back to the starting value.

Table B1 shows all the numbers along the 20 corresponding time steps, in particular the resulting allocation and gains and losses at every time step, if we strictly apply PAR.

The net gain over a full cycle turns out to equal 10 units. The pattern of the gains and losses at each point of time is presented in Figure B3.

The point of this exercise is to demonstrate that gains exceed the losses slightly but systematically. This is revealed by considering carefully the length of the bars.

A standard question is how PAR performs, if the 'true' intrinsic value differs from our perception. Hence, we assume in a second example that we underestimate the intrinsic value persistently by two units (see Figure B4 and B5)

In this case, the perceived vps determine the allocation. Moreover, Table B2 shows all the numbers.

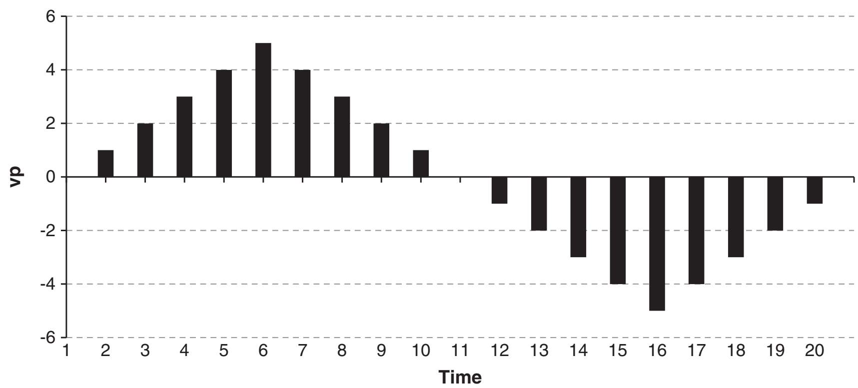

Figure B2: vp evolution with 'triangle sine' shape.
Source: proprietary.

Interestingly, the net gain over a full cycle turns out unchanged, that is, 10 units again.

However, the pattern of the individual gains and losses is quite different from the pattern evidenced in the previous example. Overall, they have a wider dispersion, resulting in a smaller information ratio. In the first example, the information ratio equals 0.78, while it is 0.64 in the second.

To sum up, being off systematically in terms of valuation does not break the model.

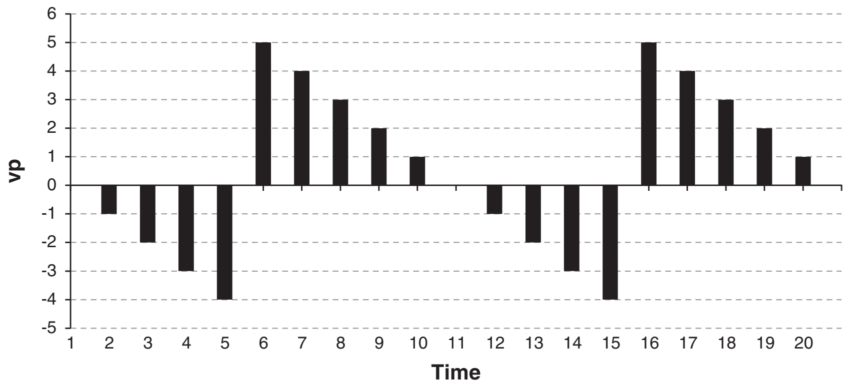

Figure B3: Gains and losses commensurate with Figure B2.
Source: proprietary.

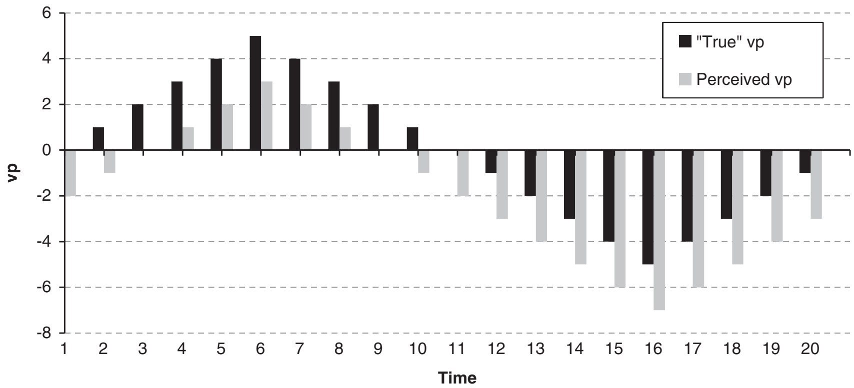

Figure B4: vp evolution with ‘triangle sine’ shape, vp underestimated systematically.
Source: proprietary.

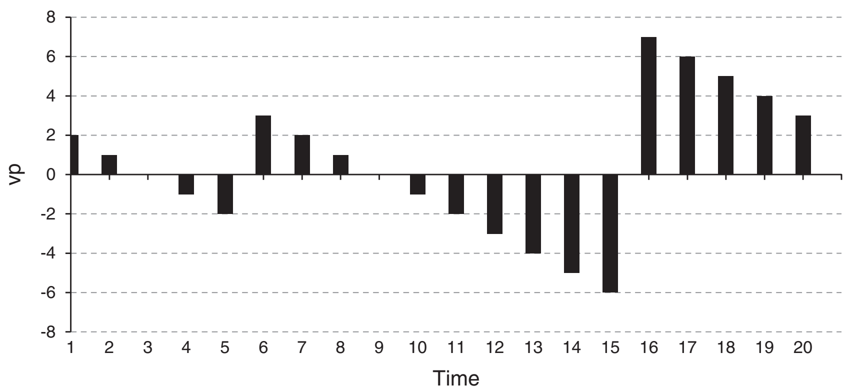

Figure B5: Gains and losses commensurate with Figure B4.
Source: proprietary.

Table B1: Numbers commensurate with Figure B2

Time	*vp*	*Pos*	*Δ(VP)*	*Gain*
1	0	0	1	0
2	1	1	1	−1
3	2	2	1	−2
4	3	3	1	−3
5	4	4	1	−4
6	5	5	−1	5
7	4	4	−1	4
8	3	3	−1	3
9	2	2	−1	2
10	1	1	−1	1
11	0	0	−1	0
12	−1	−1	−1	−1
13	−2	−2	−1	−2
14	−3	−3	−1	−3
15	−4	−4	−1	−4
16	−5	−5	1	5
17	−4	−4	1	4
18	−3	−3	1	3
19	−2	−2	1	2
20	−1	−1	1	1
Total	**10**	—	—	—

Note: Pos, Position.
Source: proprietary.

Table B2: Numbers commensurate with Figure B4

Time	*'True' vp*	*Perceived vp*	*Pos*	*Δ(VP)*	*Gain*
1	0	−2	−2	1	2
2	1	−1	−1	1	1
3	2	0	0	1	0
4	3	1	1	1	−1
5	4	2	2	1	−2
6	5	3	3	−1	3
7	4	2	2	−1	2
8	3	1	1	−1	1
9	2	0	0	−1	0
10	1	−1	−1	−1	−1
11	0	−2	−2	−1	−2
12	−1	−3	−3	−1	−3
13	−2	−4	−4	−1	−4
14	−3	−5	−5	−1	−5
15	−4	−6	−6	−1	−6
16	−5	−7	−7	1	7
17	−4	−6	−6	1	6
18	−3	−5	−5	1	5
19	−2	−4	−4	1	4
20	−1	−3	−3	1	3
Total	**10**	—	—	—	—

Note: Pos, Position.
Source: proprietary.

It just makes the result less efficient. Again, the indispensable prerequisite is mean reversion. However, the impact of mean reversion is not pivotally affected by not 'hitting' the intrinsic value exactly.

APPENDIX C

Table C1: Input covariance matrix

EQ US	15.3%	1.00	0.72	0.77	0.69	0.67	0.73	0.74	0.69	0.40	0.35	0.35	0.32	0.30	0.35	0.36	0.40	0.38	0.52	0.46	0.23	0.31
EQ UK	15.9%	0.72	1.00	0.68	0.60	0.58	0.65	0.66	0.61	0.31	0.37	0.31	0.28	0.26	0.28	0.28	0.30	0.29	0.43	0.38	0.17	0.24
EQ EMU	17.6%	0.77	0.68	1.00	0.64	0.62	0.69	0.70	0.65	0.33	0.34	0.38	0.34	0.28	0.32	0.32	0.33	0.31	0.46	0.41	0.18	0.26
EQ SWI	16.7%	0.69	0.60	0.64	1.00	0.56	0.62	0.61	0.57	0.29	0.28	0.29	0.35	0.25	0.27	0.27	0.29	0.28	0.41	0.36	0.16	0.23
EQ JAP	20.0%	0.67	0.58	0.62	0.56	1.00	0.59	0.59	0.56	0.28	0.27	0.28	0.26	0.34	0.28	0.26	0.28	0.27	0.40	0.35	0.15	0.22
EQ AUS	16.7%	0.73	0.65	0.69	0.62	0.59	1.00	0.67	0.62	0.31	0.30	0.31	0.29	0.27	0.36	0.29	0.31	0.30	0.44	0.39	0.17	0.25
EQ CAN	17.0%	0.74	0.66	0.70	0.61	0.59	0.67	1.00	0.63	0.31	0.29	0.31	0.29	0.26	0.28	0.36	0.31	0.30	0.44	0.39	0.17	0.25
EQ EMA	19.1%	0.69	0.61	0.65	0.57	0.56	0.62	0.63	1.00	0.29	0.28	0.29	0.27	0.25	0.26	0.27	0.29	0.28	0.41	0.36	0.16	0.23
BD 10Y US	7.4%	0.40	0.31	0.33	0.29	0.28	0.31	0.31	0.29	1.00	0.82	0.77	0.71	0.66	0.85	0.90	0.99	0.93	0.72	0.63	0.76	0.76
BD 10Y UK	8.0%	0.35	0.37	0.34	0.28	0.27	0.30	0.29	0.28	0.82	1.00	0.87	0.77	0.62	0.79	0.79	0.82	0.78	0.61	0.53	0.62	0.65
BD 10Y EMU	7.4%	0.35	0.31	0.38	0.29	0.28	0.31	0.31	0.29	0.77	0.87	1.00	0.87	0.66	0.81	0.78	0.77	0.73	0.59	0.51	0.57	0.61
BD 10Y CH	6.4%	0.32	0.28	0.34	0.35	0.26	0.29	0.29	0.27	0.71	0.77	0.87	1.00	0.60	0.72	0.70	0.71	0.67	0.54	0.47	0.52	0.56
BD 10Y JAP	7.1%	0.30	0.26	0.28	0.25	0.34	0.27	0.26	0.25	0.66	0.62	0.66	0.60	1.00	0.68	0.61	0.65	0.62	0.50	0.44	0.48	0.52
BD 10Y AUS	8.6%	0.35	0.28	0.32	0.27	0.28	0.36	0.28	0.26	0.85	0.79	0.81	0.72	0.68	1.00	0.80	0.85	0.81	0.62	0.54	0.64	0.67
BD 10Y CAN	8.2%	0.36	0.28	0.32	0.27	0.26	0.29	0.36	0.27	0.90	0.79	0.78	0.70	0.61	0.80	1.00	0.89	0.85	0.65	0.57	0.68	0.70
BD 5Y US	4.6%	0.40	0.30	0.33	0.29	0.28	0.31	0.31	0.29	0.99	0.82	0.77	0.71	0.65	0.85	0.89	1.00	0.97	0.72	0.63	0.76	0.83
BD 2Y US	2.3%	0.38	0.29	0.31	0.28	0.27	0.30	0.30	0.28	0.93	0.78	0.73	0.67	0.62	0.81	0.85	0.97	1.00	0.68	0.60	0.72	0.94
HY US	7.5%	0.52	0.43	0.46	0.41	0.40	0.44	0.44	0.41	0.72	0.61	0.59	0.54	0.50	0.62	0.65	0.72	0.68	1.00	0.53	0.52	0.57
BD EM	8.1%	0.46	0.38	0.41	0.36	0.35	0.39	0.39	0.36	0.63	0.53	0.51	0.47	0.44	0.54	0.57	0.63	0.60	0.53	1.00	0.46	0.50
RB 10Y US	5.1%	0.23	0.17	0.18	0.16	0.15	0.17	0.17	0.16	0.76	0.62	0.57	0.52	0.48	0.64	0.68	0.76	0.72	0.52	0.46	1.00	0.60
SB US	0.4%	0.31	0.24	0.26	0.23	0.22	0.25	0.25	0.23	0.76	0.65	0.61	0.56	0.52	0.67	0.70	0.83	0.94	0.57	0.50	0.60	1.00

APPENDIX D

Table D1: Mutual information coefficients

EQ US	0.00	0.08	0.06	0.07	0.08	0.07	0.07	0.06	0.09	0.10	0.10	0.09	0.09	0.10	0.10	0.08	0.08	0.09	0.09	0.09	0.08
EQ UK	0.08	0.00	0.08	0.08	0.08	0.08	0.07	0.07	0.09	0.09	0.09	0.09	0.09	0.10	0.09	0.08	0.08	0.09	0.09	0.09	0.08
EQ EMU	0.06	0.08	0.00	0.07	0.08	0.07	0.07	0.06	0.08	0.08	0.08	0.08	0.08	0.09	0.08	0.07	0.07	0.08	0.08	0.08	0.07
EQ SWI	0.07	0.08	0.07	0.00	0.08	0.07	0.08	0.06	0.09	0.10	0.09	0.08	0.09	0.10	0.09	0.08	0.08	0.09	0.09	0.09	0.08
EQ JAP	0.08	0.08	0.08	0.08	0.00	0.07	0.08	0.07	0.08	0.09	0.09	0.08	0.08	0.09	0.09	0.08	0.07	0.08	0.09	0.08	0.08
EQ AUS	0.07	0.08	0.07	0.07	0.07	0.00	0.07	0.06	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.08	0.08	0.09	0.10	0.09	0.08
EQ CAN	0.07	0.07	0.07	0.08	0.08	0.07	0.00	0.06	0.08	0.09	0.09	0.08	0.08	0.09	0.09	0.08	0.07	0.08	0.09	0.08	0.07
EQ EMA	0.06	0.07	0.06	0.06	0.07	0.06	0.06	0.00	0.07	0.07	0.07	0.06	0.07	0.07	0.07	0.06	0.06	0.07	0.07	0.06	0.06
BD 10Y US	0.09	0.09	0.08	0.09	0.08	0.09	0.08	0.07	0.00	0.14	0.14	0.14	0.14	0.14	0.14	0.09	0.09	0.17	0.18	0.14	0.13
BD 10Y UK	0.10	0.09	0.08	0.10	0.09	0.09	0.09	0.07	0.14	0.00	0.14	0.14	0.14	0.14	0.14	0.13	0.11	0.16	0.17	0.14	0.13
BD 10Y EMU	0.10	0.09	0.08	0.09	0.09	0.09	0.09	0.07	0.14	0.14	0.00	0.14	0.14	0.14	0.14	0.14	0.12	0.17	0.17	0.14	0.13
BD 10Y CH	0.09	0.09	0.08	0.08	0.08	0.09	0.08	0.06	0.14	0.14	0.14	0.00	0.15	0.14	0.14	0.15	0.12	0.17	0.17	0.14	0.13
BD 10Y JAP	0.09	0.09	0.08	0.09	0.08	0.09	0.08	0.07	0.14	0.14	0.14	0.15	0.00	0.15	0.14	0.15	0.13	0.16	0.17	0.15	0.14
BD 10Y AUS	0.10	0.10	0.09	0.10	0.09	0.09	0.09	0.07	0.14	0.14	0.14	0.14	0.15	0.00	0.14	0.13	0.11	0.16	0.17	0.14	0.13
BD 10Y CAN	0.10	0.09	0.08	0.09	0.09	0.09	0.09	0.07	0.14	0.14	0.14	0.14	0.14	0.14	0.00	0.13	0.11	0.16	0.17	0.14	0.13
BD 5Y US	0.08	0.08	0.07	0.08	0.08	0.08	0.08	0.06	0.09	0.13	0.14	0.15	0.15	0.13	0.13	0.00	0.08	0.17	0.18	0.15	0.15
BD 2Y US	0.08	0.08	0.07	0.08	0.07	0.08	0.07	0.06	0.09	0.11	0.12	0.12	0.13	0.11	0.11	0.08	0.00	0.15	0.16	0.12	0.24
HY US	0.09	0.09	0.08	0.09	0.08	0.09	0.08	0.07	0.17	0.16	0.17	0.17	0.16	0.16	0.16	0.17	0.15	0.00	0.17	0.17	0.16
BD EM	0.09	0.09	0.08	0.09	0.09	0.10	0.09	0.07	0.18	0.17	0.17	0.17	0.17	0.17	0.17	0.18	0.16	0.17	0.00	0.18	0.17
RB 10Y US	0.09	0.09	0.08	0.09	0.08	0.09	0.08	0.06	0.14	0.14	0.14	0.14	0.15	0.14	0.14	0.15	0.12	0.17	0.18	0.00	0.13
SB US	0.08	0.08	0.07	0.08	0.08	0.08	0.07	0.06	0.13	0.13	0.13	0.13	0.14	0.13	0.13	0.15	0.24	0.16	0.17	0.13	0.00